Systemic risks in CCP networks

Russell Barker, Andrew Dickinson, Alex Lipton and Rajeev Virmani propose a model for the credit and liquidity risks faced by clearing members of central counterparty clearing houses (CCPs). By considering the entire network of CCPs and clearing members, they investigate the distribution of losses to default fund contributions and contingent liquidity requirements for each clearing member

Since the financial crisis of 2007–10, the number of trades and the range of products that are cleared by central counterparty clearing houses (CCPs) have increased enormously.

There is a clear need for banks to assess any potential impact of defaults of general clearing members (GCMs) through the CCP network and, in particular, on themselves. However, understanding the risk is a challenge, since it requires understanding the contingent cashflows between a large number of agents (hundreds of GCMs and multiple CCPs): see figure 1 for an example of a real-world CCP network. Further, the interrelationship between the GCMs via the CCPs is a complex one, which requires capturing the dynamic evolution of variation margin (VM), initial margin (IM) and default fund (DF) contributions as well as porting trades in the event of a member default and allocating default losses. Further, the fact that a particular GCM's CCP activity may only represent a fraction of their broader economic activity should be captured. Although the system is too complex to analyse using analytically tractable models, it is viable to develop simulation models that capture the contingent cashflows between all agents (including those related to margining and defaults) and address the following important issues related to the broader application of central clearing to over-the-counter derivatives portfolios:

- potential systemic risks and contagion introduced by the interconnected nature of the system;
- liquidity issues driven by profit and loss (P&L), changes in margining, losses due to default and CCP recapitalisation;
- the connection between market volatility and default likelihood;
- the identification of the key points of failure; and
- the magnitude of scenarios proving sufficiently large in order for a given clearing member to incur a loss or suffer liquidity issues.

One of the novel aspects of the model proposed in this paper is the fact that it considers the entire network of CCPs and GCMs – which, given its size and complexity, is somewhat challenging – and yields some important insights. There are material cross-risks between the default of GCMs and market volatility that must be captured in order to realistically assess default losses and contingent liquidity requirements. Our results do not support the fear that a move from bilateral clearing to central clearing of OTC derivatives poses a significant threat of contagion through the CCPs, which is primarily attributable to the magnitude of risks being a comparatively small proportion of the capital held by the diversified financial institutions dominating the CCP membership.

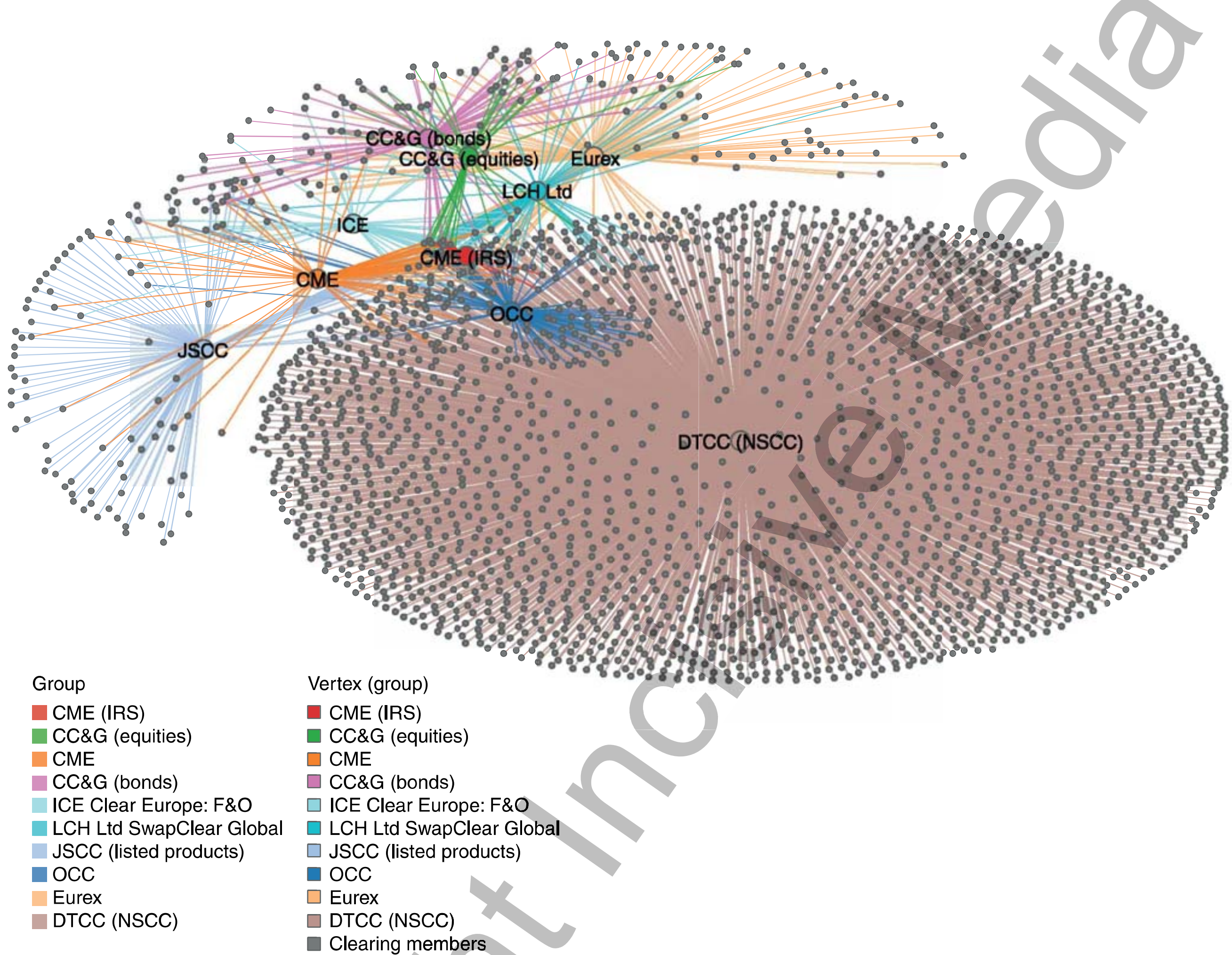
A wide variety of models has been developed in order to quantify the potential exposure of CCPs. These models can be divided into three main categories: statistical, optimisation and option-pricing-based models. Statistical models typically assume simple underlying dynamics, such as geometric Brownian motion (GBM), and derive the probability for the IM to be exceeded within a given time horizon. Optimisation models, as their name suggests, try to set margins in a way that achieves an appropriate balance between the resilience of CCPs and the costs to their members. Option-pricing-based models capitalise on the fact that the exposure profile of a CCP is approximately equivalent to the payoff of a strangle, representing a combination of a call and a put option. A GCM has a theoretical opportunity to default strategically if the contract loses more value than the posted IM.

Since the most important problem from the CCP standpoint is to analyse losses conditional on exceeding margin, extreme value theory (EVT) is well suited for this purpose. While it is relatively easy to use EVT to set up margins for a single contract, it is much more difficult at a portfolio level; hence, CCPs tend not to use EVT directly. Accordingly, in many cases CCPs use either the Standard Portfolio Analysis of Risk (Span) methodology (see Kupiec 1994) or value-at-risk methodology (see Barone-Adesi, Giannopoulos & Vosper 2002).

Several recent contributions to studying systemic risk in CCPs are worth mentioning. Duffie & Zhu (2011) discuss the premise that the central clearing of derivatives can substantially reduce counterparty risk, while Glasserman, Moallemi & Yuan (2014) consider systemic risks in markets cleared by multiple CCPs. Borovkova & El Mouttalibi (2013) analyse systemic risk in CCPs by utilising a network approach and conclude that stricter capital requirements have a clearer and stronger positive impact on the system than mandatory clearing through CCPs. Elouerkhaoui (2015) and Crépey (2015) develop a method for calculating credit value adjustment (CVA) for CCPs using a collateralised debt obligation (CDO) pricing approach by defining the payoff of the CCP's waterfall and using the Marshall-Olkin correlation model to compute it. Cumming & Noss (2013) assess the adequacy of CCPs' default resources from the perspective of the Bank of England, and argue the best way to model a CCP's exposure to a single GCM in excess of its IM is by applying EVT. Finally, Murphy & Nahai-Williamson (2014) discuss approaches to the analysis of CCP DF adequacy.

Although a lot of advances have been made in the recent literature, we feel some of the most important features of the CCP universe have been missed. The first is the feedback mechanism that intrinsically links GCM default, market turbulence and liquidity calls on market participants. The second is the individual nature of different clearing members, from large diverse financial institutions, where markets will make up a minority of their business, to proprietary funds, for which a default event will be driven purely by margin calls on cleared trades. The third is the interconnectedness of the CCPs themselves, which means it is important to model the network in its entirety. Finally, it is important to model the changes in IM and DF requirements as the system evolves; this is particularly important when modelling liquidity considerations.

1 An example network of CCPs (coloured circles) and GCMs (black dots), with corresponding coloured edges denoting membership



Affiliation graph of 10 largest CCPs by DF. ‘F&O’ denotes futures and options.

In our approach, we use the minimum amount of information necessary to analyse the risk of contagion or a liquidity crunch in the CCP framework, but we still build a realistic simulation of what might occur in a stressed situation. For each GCM and CCP, we need to model the loss over IM and DF if that GCM were to default in a specific market scenario as well as how to distribute that loss to other GCMs. We also need to be able to model the circumstances of a GCM default given a market scenario and link this to the reduction in the GCM’s capital due to losses on a number of CCPs.

In this paper, we develop a simulation framework to investigate the risks associated with central clearing. The paper is structured as follows. First, we discuss margining and its modelling. Next, we present the process by which we generate the portfolios of clearing members given the partial information a particular GCM possesses. Then, we present the simulation of the underlying market variables and the feedback mechanism used to generate realistic co-dependence between volatilities and defaults. We present numerical results in the penultimate section and conclude with the ultimate section.

Margin calculation

CCPs set up extensive processes to manage the default of any GCM; this includes requiring their GCMs post an IM and a DF contribution, along with a VM, in order to cover the mark-to-market (MtM) moves of the exposures, together with a risk waterfall process that stipulates how any eventual losses would be distributed among the defaulting clearing member, the non-defaulting members and the CCP itself. Given a set of market data and a portfolio of trades, we need to be able to calculate the VM, IM and DF for the total set of GCM portfolios on a given CCP.

We represent the state of the market at time t by:

$$X(t) = (X_1(t), \dots, X_n(t))^{\mathrm{T}}$$

where $X_i(t)$ represents a financial quantity such as a par swap, spot foreign exchange rate, credit spread, etc. We describe the generation of these scenarios below.

The incremental VM called over the time interval $[t_i, t_{i+1}]$ for the portfolio held by GCM_k, with CCP_j, is given by the change in MtM:

$$\mathrm{VM}^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t_{i+1}) - \mathrm{VM}^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t_i) = \sum_{\phi \in \Phi^{\mathrm{CCP}_j}_{\mathrm{GCM}_k}} V_\phi(\boldsymbol{X}(t_{i+1}), t_{i+1}) - V_\phi(\boldsymbol{X}(t_i), t_i)$$

where the summation is over all trades, ϕ, in the portfolio that GCM_k holds with CCP_j at time t_i, and $V_\phi(\boldsymbol{X}(t), t)$ is the value of trade ϕ at time t in market state $\boldsymbol{X}(t)$.

We need to construct a fast method for calculating the IM of GCM_k on CCP_j, based on its portfolio and the market at time t_i. In our model, we used the specifications given by the individual CCPs for calculating both IM and DF. These can be significantly different across CCPs, so it is important to use the methods as prescribed by the CCPs themselves. Generally, the IM is dominated by a VAR/conditional VAR (CVAR) component of the portfolio over a set of market changes, derived from a historic time period that is supplemented by a set of deterministic add-ons for liquidity, basis risk, etc. We split the IM into a VAR/CVAR component and a set of add-ons. In practice, the VAR/CVAR component is calculated across the portfolio losses as follows:

- a set of scenarios is created by looking at a set of five-day change-in-market data over some historic period;
- these changes are weighted by a multiple of the ratio of current to historic realised volatility;
- new scenarios are created by using the current market data and the set of market changes to calculate the five-day loss on the portfolio for each scenario; and
- the VAR or CVAR is calculated by using these.

We calculate separately the VAR/CVAR component using regression against a collection of representative portfolios. We then apply the add-ons deterministically. First, we evaluate the IM on a set of portfolios that are sufficiently small so as not to incur any add-ons, and the IM on these is calculated using the full IM calculation process. Second, the IM for the current portfolio is calculated as:

$$\mathrm{IM}^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t) = \mathrm{VAR}(\{\boldsymbol{X}(u)\}_{u \leqslant t}; \boldsymbol{a}(t)) + \mathrm{AddOn}(t) \qquad (1)$$

where $\boldsymbol{a}(t) = (a_1(t), \dots, a_n(t))^{\mathrm{T}}$ represents the regression coefficients of the portfolio held at time t against the small benchmark portfolios for which IM has been calculated in the first step. $\mathrm{VAR}(\{\boldsymbol{X}(u)\}_{u \leqslant t}; \boldsymbol{a}(t))$ represents the VAR component of the IM for a portfolio represented by these regression coefficients and adjusted for new simulated market data. This takes into account (a) the change in the ratio of current market to historic volatilities, which can be estimated by keeping track of the historic multiplier and adjusting appropriately, and (b) whether the new scenario creates a loss large enough to replace one of the VAR or CVAR elements.[1] The add-on is a deterministic function of the underlying portfolio specified by each CCP. To take (b) above into account, we heuristically model the historical loss distribution of the portfolio by a one-parameter family of distributions (such as the family of centred Gaussian distributions), fitted to the level of the IM at the previous time step, and update according to whether the realisation of the loss over the current time step is sufficiently extreme.

The total default fund at time t is given by a 'Cover 2' principle:

$$\mathrm{DF}_{\mathrm{CCP}_j}(t) = \max_{\sigma \in S_j} \max_{k \neq l} \Big[\mathrm{LOIM}^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t, \sigma) + \mathrm{LOIM}^{\mathrm{GCM}_l}_{\mathrm{CCP}_j}(t, \sigma) - K_{\mathrm{CCP}_j}\Big]^+$$

where the maximum is over all stress scenarios, S_j, for CCP_j and distinct pairs of (surviving) GCMs, GCM_k, GCM_l, while the loss over IM, $\mathrm{LOIM}^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t, \sigma)$, is given by:

$$\mathrm{LOIM}^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t, \sigma) = \Bigg[\sum_{\phi \in \Phi^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t)} (V_\phi(t, \boldsymbol{X}(t)) - V_\phi(t, \boldsymbol{X}^\sigma(t))) + \mathrm{IM}^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t)\Bigg]^+$$

where the summation is over trades, ϕ, in the portfolio $\Phi^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t)$, and $V_\phi(t, \boldsymbol{X}^\sigma(t))$, $V_\phi(t, \boldsymbol{X}(t))$ are the values of ϕ at time t in market state $\boldsymbol{X}(t)$, with and without the stress scenario, σ, applied. We remark that, although CCPs define a large number of stress scenarios, typically, there are comparatively few 'binding' scenarios, and the scenarios may be replaced with a significantly smaller subset. This was tested by examining thousands of actual and randomly generated realistic sets of portfolios to see which scenarios generated the largest losses. A significant number of the scenarios used by the CCP were never binding.

The method we used to allocate DF among the members was as prescribed by the CCPs themselves.

GCM portfolios

In this section, we adopt the perspective of a particular (but arbitrary) clearing member and discuss how it may assess its risks by making use of the partial information available to it. It is not uncommon for a banking group to have multiple subsidiaries, each of which is a distinct clearing member. We write $\{\mathrm{GCM}_k\}_{k \in K}$ for these subsidiaries, whose positions with each CCP are known to the group. To aid discussion, let us introduce a name for this banking group: 'XYZ Bank'. We refer to the gross notional as the total notional over long and short positions, while the net notional is the difference. The reader should note risks are ultimately determined by net positions; hence, net notionals are of primary concern. However, gross notionals provide useful and important information on accumulated historical volumes.

Although XYZ is potentially exposed to the positions of other members in the event of their default, the positions of other members are unknown. However, gross notionals for certain categories of derivatives, aggregated across all members, are published by the CCP, where the categories are typically discriminated by the type of trade, currency and tenor. For example, a CCP may publish the aggregate gross notional for fixed versus six-month Euribor swaps for tenors in the range two to five years (alongside other aggregates). We wish to make use of these aggregate gross notionals as a measure of the relative scale of the exposures of the CCP in different

[1] *We assume a VAR/CVAR scenario does not roll off the historic period specified; this is a reasonable assumption, as most CCPs include the 2007–12 period and have ensured this will not drop out of the historic period in the near future, and the time horizons for our simulations are around one year.*

trade types, currencies and tenors. For this reason, it is appropriate to align our methodology to the categorisation used by the CCPs when reporting aggregate gross notionals. Below, we will fix a particular category $\pi \in \Pi$, where Π represents the set of all categories used by the CCP to disclose aggregate gross notionals.

To assess XYZ's risks, we propose a randomisation scheme to explore the space of valid configurations of the unknown positions of other GCMs, subject to the constraints of reproducing the known information: values related to XYZ's positions and the aggregate gross notionals published by the CCPs. To each GCM, k, we assign a rank $J_k \in \{1, \dots, n\}$, based upon data sourced from publicly available information, such as financial statements. We then fit a two-parameter exponential distribution to the gross notional of the members, motivated by the analysis of Murphy & Nahai-Williamson (2014):

$$\sum_{k=1}^{n} \beta^{\pi} \exp(-\alpha^{\pi} J_k) = N^{\pi}$$

$$\sum_{k \in K} \beta^{\pi} \exp(-\alpha^{\pi} J_k) = N_K^{\pi}$$

where K is the set of indexes of XYZ's members, while N^{π} and N_K^{π} are the gross notional amounts for category π aggregated over all members and XYZ's members, respectively. The system is solved numerically for $\alpha^{\pi}, \beta^{\pi}$, and we abbreviate the fitted net notional for k as:

$$N_k^{\pi} = \beta^{\pi} \exp(-\alpha^{\pi} J_k)$$

We generate randomised net notionals, $\{\Delta_k^{\pi}\}_{k=1}^{n}$, such that long and short positions net:

$$\sum_{k=1}^{n} \Delta_k^{\pi} = 0$$

and:

$$\begin{aligned} \Delta_k^{\pi} &\in [-RN_k^{\pi}, RN_k^{\pi}] \quad \text{for all } k \\ \Delta_k^{\pi} &= \delta_k^{\pi} \quad \text{for all } k \in K \end{aligned} \tag{2}$$

where $R \in [0, 1]$ is a parameter controlling the relative size of the net and gross positions, and $\{\delta_k\}_{k \in K}$ are the known net positions for XYZ. R may be thought of as a proportional trading delta limit. For reasons of parsimony, we assume the parameter does not depend upon the GCM, k, or the product classification, π. Of course, this assumption could be weakened if appropriate.

Introduce the negative total sum of all known positions, $\bar{\Delta}^{\pi} = -\sum_{k \in K} \delta_k^{\pi}$; define the ratio $r^{\pi} = \bar{\Delta}^{\pi} / \sum_{k \notin K} N_k^{\pi}$; and use it to proportionally allocate $\bar{\Delta}^{\pi}$ among GCMs with $k \notin K$, $\bar{\Delta}_k^{\pi} = r^{\pi} N_k^{\pi}$. Without loss of generality, we assume $|r^{\pi}| < R$. For each $k \notin K$, consider the interval $I_k^{\pi} = [-(R - |r^{\pi}|), (R - |r^{\pi}|)]$ and generate independent random numbers u_k^{π}, uniformly distributed on I_k^{π}. Define the following quantities:

$$U^{\pi} = \sum_{k \notin K} u_k^{\pi} N_k^{\pi}, \qquad V^{\pi} = \sum_{k \notin K} u_k^{\pi} N_k^{\pi} \chi_{u_k^{\pi} U > 0}, \qquad W^{\pi} = \frac{U^{\pi}}{V^{\pi}}$$

where χ is the indicator function. Since u_k^{π} possesses a density, it is clear $V^{\pi} \neq 0$ almost surely, W^{π} is well defined and $0 < W^{\pi} \leqslant 1$. Define the net position of the kth GCM as follows:

$$\begin{aligned} \Delta_k^{\pi} &= \bar{\Delta}_k^{\pi} + u_k^{\pi}(1 - W^{\pi} \chi_{u_k^{\pi} U^{\pi} > 0}) N_k^{\pi} \\ &= (r^{\pi} + u_k^{\pi}(1 - W^{\pi} \chi_{u_k^{\pi} U^{\pi} > 0})) N_k^{\pi} \end{aligned}$$

In words, we proportionally reduce positions for GCMs with u_k^{π} that have the same sign as U^{π}, and we keep positions for other GCMs fixed. A simple calculation yields:

$$\sum_{k \notin K} \Delta_k^{\pi} = \bar{\Delta}^{\pi} + U^{\pi} - \frac{U^{\pi}}{V^{\pi}} V^{\pi} = \bar{\Delta}^{\pi}$$

$$\begin{aligned} |\Delta_k^{\pi}| &= |\Delta_k^{\pi} - \bar{\Delta}_k^{\pi} + \bar{\Delta}_k^{\pi}| \leqslant |\Delta_k^{\pi} - \bar{\Delta}_k^{\pi}| + |\bar{\Delta}_k^{\pi}| \\ &\leqslant |u_k| + |r^{\pi}| N_k^{\pi} \leqslant (R - |r^{\pi}| + |r^{\pi}|) N_k^{\pi} = RN_k^{\pi} \end{aligned}$$

so both conditions for Δ_k^{π}, $k \notin K$, are satisfied.

Scenario generation

In this section, we describe the model for the underlying market variables that is used to evolve the system of GCMs, given the initial positions generated according to the scheme presented above. We wish to ensure the model is rich enough to support jumps, allowing for comparatively large changes on short time scales, including systemic jumps that affect all market variables simultaneously. We reflect that periods of high default rates will be accompanied by high market volatility (as was observed during the crisis). For these reasons, we will propose a regime-switching model, with regimes driven by the number and size of realised defaults.

Regime-dependent drivers. For each GCM_k, we introduce a weight $w_k > 0$, which represents the financial significance of the GCM to the others and is normalised so $\sum_k w_k = 1$. Practically, we set these weights to be proportional to the balance sheet assets of each GCM.

We introduce a stress indicator, Ξ_t, by:

$$\Xi_t = \sum_k w_k \mathrm{e}^{-\theta_{\Xi}\{t - \tau_k\}} \chi_{\tau_k < t}$$

which yields a value between 0 and 1, representing the materiality weighted defaults prior to time t; τ_k is the default time of GCM_k, and θ_{Ξ} represents a rate of mean reversion from a stress state to equilibrium. It will be set as 1 in what follows.

We introduce some thresholds $0 < m_1 < m_2 < \dots < m_S = 1$ that determine the stress state, and we define the stress state process, $\xi_t^{\boldsymbol{m}}$, which takes its value in $\{1, \dots, S\}$, by:

$$\xi_t^{\boldsymbol{m}} = \begin{cases} 1, & \Xi_t \leqslant m_1 \\ i, & m_{i-1} < \Xi_t \leqslant m_i, \ i \geqslant 2 \end{cases}$$

We introduce $1 = \Lambda_1 < \dots < \Lambda_S$, which represents volatility multipliers in each of the S stress states.

We consider Brownian drivers, $W_t^{\xi^{\boldsymbol{m}}}$, with regime-dependent volatilities, so, conditional on the stress state $\xi^{\boldsymbol{m}}$, the volatility of $W_t^{\xi^{\boldsymbol{m}}}$ is $\xi_t^{\boldsymbol{m}}$:

$$\mathrm{d}\langle W^{\xi^{\boldsymbol{m}}}, W^{\xi^{\boldsymbol{m}}} \rangle_t = \Lambda_{\xi_t^{\boldsymbol{m}}}^2 \, \mathrm{d}t$$

Similarly, we consider a regime-switching compound Poisson driver, $N_t^{\xi^{\boldsymbol{m}}}$, where, conditional on the value of $\xi_t^{\boldsymbol{m}}$, the intensity of the Poisson process is $\lambda \Lambda_{\xi_t^{\boldsymbol{m}}}$. The jump distribution is as proposed in Inglis *et al* (2008), being equal in distribution to the random variable:

$$\mathrm{e}^{Z} - 1$$

where $Z \sim N(\mu, \sigma)$. The Poisson driver is compensated so as to be a martingale:

$$[N_u^{\xi^m} - N_t^{\xi^m} \mid \quad t] = 0$$

for $u \geqslant t$.

We remark that the underlying simulation generates the regime-dependent Wiener and Poisson processes via a numerical scheme that is based on superposition of the standard Wiener and Poisson processes over a regime-dependent number of states. This ensures comparability across different regimes for a particular realisation.

We describe the usage of these drivers below. Note losses on default and liquidity drains are primarily driven by increments in the value of portfolios over short time horizons; for this reason, questions related to measure-dependent drifts and second-order convexity adjustments are neglected. We assume all processes are sensitive to a common regime-dependent Poisson process, which we denote by N_t^{sys,ξ^m}.

■ **Rates process.** Interest rates in the ith economy are simulated by analogy to a simple two-factor Hull-White model:

$$\mathrm{d}r_t^i = \mathrm{d}\phi_t^i + \mathrm{d}X_t^1 + \mathrm{d}X_t^2 + \beta_i r_{t-}^i \,\mathrm{d}N_t^{\text{sys},\xi^m} + r_{t-}^i \,\mathrm{d}N_t^{i,\xi^m}$$

where ϕ_t^i is deterministic (used to fit the initial term structure); X_t^1, X_t^2 are Ornstein-Uhlenbeck processes, driven by regime-dependent correlated Wiener processes W_t^{1,ξ^m}, W_t^{2,ξ^m} of the form considered above, which are used to control the relative volatility of rates of different tenors and intra-curve spread volatilities; N_t^{sys,ξ^m}, N_t^{i,ξ^m} are compound, compensated, regime-dependent Poisson processes of the form considered above, representing a systemic (respectively idiosyncratic) jump process; β_i represents the sensitivity of rates to the systemic jump process; and $t-$ denotes the left-hand limit. As described below, we calibrate the parameters to historical, rather than implied, market data.

Market observables (swap rates, Libor rates) are calculated from the state (X_t^1, X_t^2) by applying the functional forms derived from the corresponding (affine) two-factor Hull-White model (without feedback and jump terms).

The above model may be viewed as a minimally complex model that has the following key features: (a) jumps, so as to allow extremal events over short time periods; (b) regime-dependent volatilities and intensities, so as to capture the natural increasing co-dependence with defaults; (c) intra-curve spread volatility, so as to ensure a reasonable P&L distribution for delta-neutral steepener/flattener positions.

■ **Forex process.** We model the spot forex analogously, with the spot forex between the ith and jth being governed by:

$$\mathrm{d}X_t^{i,j} = \sigma_t^{i,j} X_t^{i,j} \,\mathrm{d}W_t^{i,j,\xi^m} + \beta_{i,j} X_{t-}^{i,j} \,\mathrm{d}N_t^{\text{sys},\xi^m} + X_{t-}^{i,j} \,\mathrm{d}N_t^{i,j,\xi^m} \tag{3}$$

where $\sigma_t^{i,j}$ is a deterministic function of time, $\beta_{i,j}$ is the sensitivity to the systemic jump process N_t^{sys,ξ^m} and N_t^{i,j,ξ^m} is an idiosyncratic jump process independent of all else conditional on ξ^m.

■ **Non-CCP asset process.** We model the non-CCP assets of the kth GCM by a process of the same form as in (3):

$$\mathrm{d}A_t^k = \sigma_t^k A_t^k \,\mathrm{d}W_t^{k,\xi^m} + \beta_k A_{t-}^k \,\mathrm{d}N_t^{\text{sys},\xi^m} + A_{t-}^k \,\mathrm{d}N_t^{k,\xi^m}$$

where each parameter is analogous.

■ **Default events.** The default of each GCM is then determined by a structural model inspired by the Merton-Black-Cox model. More precisely, the default time of the kth member is determined as the hitting time of the total position of CCP-related and non-CCP related activities:

$$\tau_k = \inf\{t > 0 \colon C_t^k - C_0^k + A_t^k \leqslant B_t^k\}$$

where B_t^k is a deterministic barrier, A_t^k is as above and represents non-CCP assets and $C_t^k - C_0^k$ is the net cashflow due to CCP-related activities (see below). The barriers are calibrated numerically so as to reproduce target default probabilities.

According to the business model of the member, the contribution to the volatility of a particular member's net assets attributable to CCP-related activity, C_t, and non-CCP-related activity, A_t, may vary considerably among members. It is useful to categorise members as follows: (a) large diversified financial institutions, whose assets are dominated by non-CCP related activity, A_t; (b) large markets-driven houses, in which the trading book makes up a significant part of their business, and for which the volatility of C_t is substantial relative to that of A_t; and (c) trading houses, for which the volatility of C_t may exceed A_t.

Once the total loss for a given CCP has been calculated, we need to distribute the losses across the remaining GCMs and go through the standard waterfall process. In reality, the method applied will vary considerably between CCPs and depend on the outcome of the auction process, which is not amenable to fine modelling. Instead, we simply distribute the losses among the surviving GCMs in proportion to their IMs, as though all surviving GCMs bided equally in the auction process. Similarly, we redistribute the net positions among all surviving members, proportionally to the size of their IM.

Finally, we need to consider how to model a situation in which the losses exceed all of the CCP's buffers. Namely, the losses exceed the total DF and any 'end of the waterfall' mitigation measures the CCP has in place. We make the assumption surviving GCMs will make good the variation on the value of cleared trades up to the time of default of the CCP; then, at this point, all trades cleared on the CCP will be unwound at par. All losses will be divided in the ratio of the surviving GCM's closing IMs. As there has never been a major CCP default, it is difficult to say how realistic this resolution is. However, the authors of this paper believe it to be a reasonable and parsimonious modelling assumption.

We may summarise this mathematically as the incremental cashflows being represented by:

$$\begin{aligned} C_t^k - C_0^k = &-\sum_{\mathrm{CCP}_j} \mathrm{IM}_{\mathrm{CCP}_j}^{\mathrm{GCM}_k}(t) - \mathrm{IM}_{\mathrm{CCP}_j}^{\mathrm{GCM}_k}(0) \\ &+ \sum_{\mathrm{CCP}_j} \mathrm{VM}_{\mathrm{CCP}_j}^{\mathrm{GCM}_k}(t) - \mathrm{VM}_{\mathrm{CCP}_j}^{\mathrm{GCM}_k}(0) \\ &- \sum_{\mathrm{CCP}_j} \sum_{t_{i+1} < t} \mathrm{LossIMDF}_{\mathrm{CCP}_j}^{\mathrm{GCM}_k}(t_i, t_{i+1}) \end{aligned}$$

where the summation is over all CCPs, and $\mathrm{IM}_{\mathrm{CCP}_j}^{\mathrm{GCM}_k}(t)$, $\mathrm{VM}_{\mathrm{CCP}_j}^{\mathrm{GCM}_k}(t)$ are the IM and VM margins. $\mathrm{LossIMDF}_{\mathrm{CCP}_j}^{\mathrm{GCM}_k}(t_i, t_{i+1})$, the loss over IM and DF for a CCP_j over time interval $(t_i, t_{i+1}]$, allocated to GCM_k, is given by:

$$\mathrm{LossIMDF}^{\mathrm{GCM}_k}_{\mathrm{CCP}_j}(t_i, t_{i+1}) = \frac{\chi_{\tau_k > t}\,\mathrm{IM}^{\mathrm{CCP}_j}_{\mathrm{GCM}_k}}{\sum_l \chi_{\tau_l > t}\,\mathrm{IM}^{\mathrm{CCP}_j}_{\mathrm{GCM}_l}}\,\mathrm{LossIMDF}_{\mathrm{CCP}_j}(t_i, t_{i+1})$$

with the total loss over IM and DF for CCP_j given by

$$\mathrm{LossIMDF}_{\mathrm{CCP}_j}(t_i, t_{i+1}) = \sum_{l\,:\,\tau_l \in (t_i, t_{i+1}]} \Bigg(\sum_{\phi \in \Phi^{\mathrm{GCM}_l}_{\mathrm{CCP}_j}(t_i)} (V_\phi(t_{i+1}) - V_\phi(t_i)) + \mathrm{IM}^{\mathrm{GCM}_l}_{\mathrm{CCP}_j}(t_i) + \mathrm{DF}^{\mathrm{GCM}_l}_{\mathrm{CCP}_j}(t_i) \Bigg)^{-}$$

with the summation over those GCMs (if any) that have defaulted in the time interval $(t_i, t_{i+1}]$. The length, $t_{i+1} - t_i$, of time intervals in the simulation is aligned with the time horizon corresponding to the VAR methodology of the CCP (typically five business days).

Results

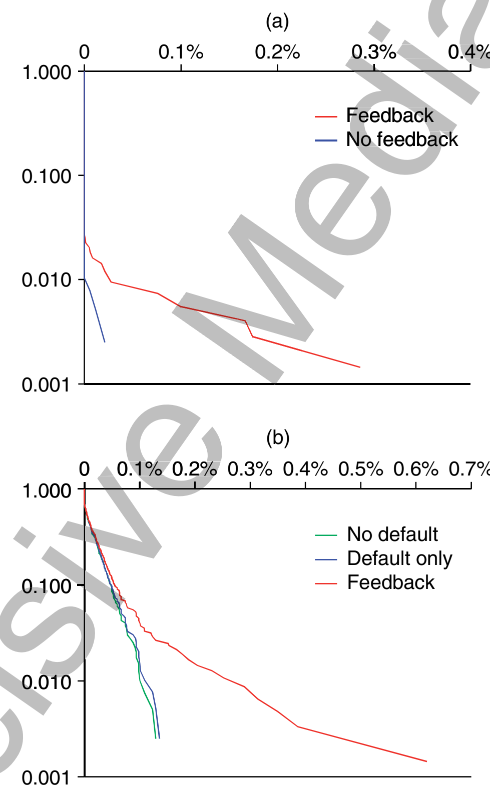
In this section, we present results for a realistic configuration of the model. We employ the perspective of a generic general clearing member, 'XYZ bank', which proxies one of the 'big four' US banks in scale but has anonymised positions that are set to be a fixed proportion of the outstanding gross notional of each of LCH.SwapClear and CME for US dollar and euro fixed-float swaps. It is supposed there are 101 clearing members, each belonging to both CCPs.

The authors accept that, due to the complexity of the CCP network, the model is of necessity large and complicated, and its calibration does require making some conservative assumptions. However, we feel the model is both realistic in terms of dynamics and robust in terms of the choices made, so we feel confident in the legitimacy of any derived results. There are several aspects requiring calibration. We give a brief overview of the techniques used to calibrate the various sections. Further details can be found in Barker, Dickinson & Lipton (2016).

The market is driven by a switching between sets of jump diffusion processes. In our example, there are two states that are differentiated by a volatility multiplier. This volatility multiplier is set to 2, which is a conservative reflection of the change in market volatility during the global financial crisis, where the volatility for rates processes rose by up to 1.5 times. The underlying processes are calibrated to the current two-year and 10-year swap rates and historic volatility. This is done in a three-stage process. First, we calibrate the dynamics without jumps analytically. Next, we use a fixed-point iteration to calibrate the model, excluding the feedback mechanism. Finally, we use the minimal entropy technique to calibrate the model's full market dynamics to the required targets.

For the largest GCMs, we use the most recent published financial statements to fit the initial levels of non-derivative assets. We then use the levels of one of the mid-size GCMs as a proxy for the rest. The authors accept that financial information is only available periodically, so it only gives a snapshot in time, but we feel this is the best information available: trying to pull further information from market-observable assets such as credit default swaps (CDSs) would just add unnecessary complexity and not give any more genuine information. This is reasonable, as it will be

2 Logarithmic plots of CCDF



(a) Ratio of losses due to default to equity (%) of XYZ bank at a time horizon of one year with and without feedback. (b) Ratio of terminal IM-initial IM to equity (%) over both CCPs of XYZ bank at a time horizon of one year for different model configurations.

the largest GCMs that contribute most to the stability of the system and, therefore, require the most accurate modelling. Each GCM has its own set of market dynamics; these are driven by a common factor, which is calibrated to the volatility of a portfolio of financial institutions, and an idiosyncratic factor, which is determined by the historic correlation of the GCM's share prices The GCM's cleared portfolios are calibrated using the portfolio generation method described above; the parameter R in (2) is used to ensure we capture the total IM and DF reported by the various CCPs and, hence, the DFs of any known GCM. Recall that, since we are using known actual portfolios where available, in this case, as we use the actual IM calculation methods employed by the various CCPs, we are guaranteed to get the correct IMs. The barriers described in the 'Default events' section are calibrated numerically to implied default probabilities.

We wish to consider the distribution at a one-year time horizon of (1) the losses due to defaults (of other GCMs and CCPs) and (2) the potential liquidity drains on XYZ bank. These distributions are scaled by the shareholder equity of XYZ bank, since we wish to size the relative significance of losses to capital buffers and understand the qualitative impact of feedback on the loss distribution. For the purposes of the example presented here, we set this to \$200 billion, approximating the magnitude of the shareholder equity of a 'big-four' US bank.

Although we employ the perspective of XYZ bank, we reiterate that we take into account the contingent cashflows between all agents in the CCP network. We study the dependence of these distributions in different configurations: defaults with the feedback-based regime-switching (‘Feedback’ in figure 2), defaults only (‘Default only’) and no defaults (‘No default’). We have made use of a minimal entropy path-reweighting algorithm to guarantee the expected stress indicator, $[\Xi_1]$, is held fixed as we change the settings of our feedback mechanism. We do this to ensure comparability of the results across configurations. The plots in figure 2 show the complementary cumulative distribution functions with the y-axis on a logarithmic scale, so, for example, a y-value of 0.01 corresponds to a 99% quantile of the loss distribution.

Figure 2(a) presents the simulated distribution of the ratio of the losses due to default (across all of XYZ’s members and CCPs) to XYZ’s shareholder equity. It demonstrates two key points. First, the effect of feedback dramatically amplifies the tail of the loss distribution due to default, which reflects the importance of capturing the natural wrong-way risk between defaults and market volatility. Second, even taking into account the interconnected and complex relationships between the agents of the CCP network, and making conservative assumptions concerning the relationship between defaults and market volatility, the scale of the losses is unlikely to threaten the survival of a well-diversified and well-capitalised financial institution.

Figure 2(b) presents the simulated distribution for the ratio of the additional aggregate IM to shareholder equity required by XYZ bank, posted to the two CCPs in the system. This captures the effect of new extremal events entering the VAR lookback period as well as potential increases in portfolio size due to the porting of defaulting members’ portfolios. The results demonstrate the significance of capturing the likely increase in volatility in stressed market conditions; further, they indicate that, in dollar terms, liquidity drains due to margining are likely to exceed losses due to default.

Conclusions

Understanding the risks associated with central clearing is technically challenging, since it requires understanding a large network of GCMs and CCPs and the complex interactions between them that are associated with margining, default fund contributions and loss waterfall structures. We have presented how, using suitable heuristics, a faithful reflection of the contingent cashflows between all of the agents in the CCP network may be simulated, and the associated risks investigated. Based on a model capturing the likely feedback between defaults and volatility, we have presented results that indicate tail losses and increased liquidity requirements require careful modelling so as to capture the substantial wrong-way risk between the volatility of market variables and defaults. Further, liquidity risks dominate those related to credit risk. This all suggests, when it comes to members assessing the risks and costs associated with their central clearing activities, their primary focus should be on funding and liquidity.

The fear the wider application of central clearing to OTC derivatives will have a destabilising effect on the financial system due to contagion effects transmitted through CCPs is not supported by our experiments. Primarily, this is attributable to (even conservative) bounds on losses due to default and contingent liquidity requirements being a small fraction of the tier 1 common equity of the diversified financial institutions that dominate CCP membership. However, the reader should note any CCP-related losses are likely to be realised precisely under the extreme circumstances where members are least able to absorb them. ■

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